

# Induced coherence with and without induced emission

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We analyze signal coherence in the setup of Wang, Zou and Mandel, where two optical downconverters have indistinct idler modes. Quantum interference, caused by indistinguishability of paths, has a visibility proportional to the transmission amplitude between idlers. Classical interference, caused by induced emission, may be complete for any finite transmission.

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The phenomenon of induced emission (that is, emission stimulated in a system by an input from another system) is well-known in laser technology [1]. It causes the phase of the amplified field to adopt the same phase as the incident locking field. It can also be used in parametric down conversion to lock the phase of the idler, and hence that of the signal (since the phase sum of the signal and idler is locked to the pump phase) [2]. If the field used to lock the idler of one downconverter (DC2 in Fig. 1) is itself the idler output of another downconverter (DC1 in Fig. 1), the two signal fields will be locked in phase also. Thus they will have (in principle) perfect first order coherence and so will interfere at the final beam splitter in Fig. 1. If there is no connection between the two downconverters, and hence no induced emission, the two signals will be incoherent, and there will be no interference. The classical explanation for this is that in parametric downconversion the phase of the signal and idler vary randomly from shot to shot, with only their sum being fixed by the pump phase.

The above arguments are completely classical. Wang, Zou and Mandel [3] (WZM) used a completely different (quantum mechanical) explanation, based on indistinguishability of paths, to explain the interference they observed in their realization of the experiment shown in Fig. 1. They did this for the very good reason that there was no induced emission in their experiment, as the downconversion rates were so low that the probability of both crystals producing a downconverted pair was negligible. Nonetheless, their analysis showed that for perfect matching of idler modes, the signal fields from DC1 and DC2 show perfect interference, while the interference is lost if the idler fields are distinguishable.

The quantum analysis used by WZM is the only correct explanation of their experiment. However, the existence of a classical theory which also reproduces these coherence features poses the following question: when is interference due to induced emission and when is it due to indistinguishability of quantum transition paths? Put another way, what, in the results of WZM, is the signature of quantum induced coherence, as distinct from classical induced emission? In this letter we show that the signature is the linear dependence of the coherence

on the transmission amplitude  $t$  from the output of idler 1 to the input of idler 2.

Before presenting our analysis, we note that the question of classical versus quantum explanations for first-order interference in parametric down conversion has arisen before [4] with reference to an experiment of Herzog *et al.* [5]. In this experiment both signal and idler fields were reflected and passed through a single downconverter a second time. Both classical and quantum arguments predict first-order interference features in the resulting fields, but also here with different magnitudes of visibility [4]. An elegant experiment with a single, but spatially extended, down-converter was recently performed, where the same discussion appears as to whether the signal and idler fields stimulate down conversion of future pump pulses further along the crystal, or whether different pulses interfere because of the indistinguishability between photons created at different times and places inside the crystal [6].

We turn now to our, fully quantum, analysis of the WZM experiment. In an appropriate limit the system can be described by four modes,  $s_1, i_1$  (the signal and idler for DC1), and  $s_2, i_2$  (the signal and idler for DC2). Consider an arbitrary operator in the Hilbert space of these four modes. The equation giving its transformation from its value  $O$  before the interaction to its value  $O'$  after the action of the downconverters and the idler transmission between DC1 and DC2 is

$$O' = U_1^\dagger U_i^\dagger U_2^\dagger O U_2 U_i U_1 \quad (1)$$

Here  $U_\mu$  for  $\mu = 1, 2$  describe the downconversion in the undepleted pump approximation. The crystals and pumps are assumed to be identical so that

$$U_\mu = \exp[-i\chi(a_{s_\mu}a_{i_\mu} + a_{s_\mu}^\dagger a_{i_\mu}^\dagger)] \quad (2)$$

where the  $a$ 's represent annihilation operators. In between the downconversions the idler from DC1 is put through a beam splitter, and becomes the idler for DC2. This is described by

$$U_t = \exp[i(\arcsin t)(a_{i_1}^\dagger a_{i_2} - a_{i_2}^\dagger a_{i_1})], \quad (3)$$

where the beam splitter transmittance  $t$  can vary between zero (where idler 2 is independent from idler 1) and unity (where idler 1 output is equal to idler 2 input).

Using Eq. (1) we easily obtain the following

$$a'_{s_1} = a_{s_1} \cosh \chi - i a_{i_1}^\dagger \sinh \chi \quad (4)$$

$$a'_{s_2} = a_{s_2} \cosh \chi - i r a_{i_2}^\dagger \sinh \chi - i t (a_{i_1}^\dagger \cosh \chi + i a_{s_1} \sinh \chi) \sinh \chi, \quad (5)$$

where  $r = \sqrt{1-t^2}$ . Since all of the initial fields are in the vacuum state it is easy to obtain the expectation values

$$\langle a_{s_1}'^\dagger a_{s_1}' \rangle = \sinh^2 \chi \quad (6)$$

$$\langle a_{s_2}'^\dagger a_{s_2}' \rangle = \sinh^2 \chi (r^2 + t^2 \cosh^2 \chi) \quad (7)$$

$$\langle a_{s_1}'^\dagger a_{s_2}' \rangle = \sinh^2 \chi t \cosh \chi \quad (8)$$

Note that the two signal modes have equal intensity only in the limit  $\chi \ll 1$ .

The maximum obtainable visibility between two fields in an experiment is given by the modulus of the first order coherence function between those fields,

$$g^{(1)}(1, 2) = \left| \langle a_1^\dagger a_2 \rangle \right| / \sqrt{\langle a_1^\dagger a_1 \rangle \langle a_2^\dagger a_2 \rangle} \quad (9)$$

In this case we find between the two final signal fields

$$g^{(1)}(1, 2) = \frac{t \cosh \chi}{\sqrt{1 + t^2 \sinh^2 \chi}}. \quad (10)$$

Noting that idler 1, before it enters the beam splitter, has the same statistics as signal 1, we can rewrite (10) in terms of the mean photon number  $\bar{n}_1 = \sinh^2 \chi$  entering the beam splitter as

$$g^{(1)}(1, 2) = t \sqrt{\frac{1 + \bar{n}_1}{1 + t^2 \bar{n}_1}}. \quad (11)$$

In this form it is easy to consider the relevant limits. The single-photon regime, which is the regime of the experiment and theory in Ref. [3], occurs for  $\bar{n}_1 \ll 1$ . That is, the probability of a downconversion at DC1 is small, and hence the probability to have downconversions at both crystals is negligible. Then the analysis of WZM applies and we expect the maximum visibility to be equal to  $t$ . This is exactly what Eq. (11) predicts.

The opposite regime is that where  $\bar{n}_1 \gg 1$ . Here there are many photons on average in all of the downconverted

beams. Thus, we could expect the classical argument to apply (although our analysis remains of course completely quantum mechanical). That is, the phase of idler 1 should lock the phase of idler 2 for any nonzero transmittance  $t$ . Again, this is reproduced by Eq. (11), which in the limit  $\bar{n}_1 \rightarrow \infty$  is equal to unity for  $t > 0$  and zero for  $t = 0$ .

For finite values of the first idler photon number output  $\bar{n}_1$ , the maximum visibility is a concave-down function of  $t$ , as shown in Fig. 2. It is evident that even photon numbers of order unity produce marked deviations from linearity. This would be interesting to observe experimentally.

To conclude, we have shown that, regardless of the number of photons involved, the first-order coherence of two signal beams is unity when one idler perfectly seeds the second, and zero when the two are independent. The difference between the quantum (single-photon) and classical (many-photon) regimes is for intermediate values of  $t$ , the transmittance of the beam splitter which transmits the first idler output into the second crystal. A linear dependence of visibility on  $t$ , as seen convincingly in Ref. [3], is the true signature of induced coherence without induced emission.

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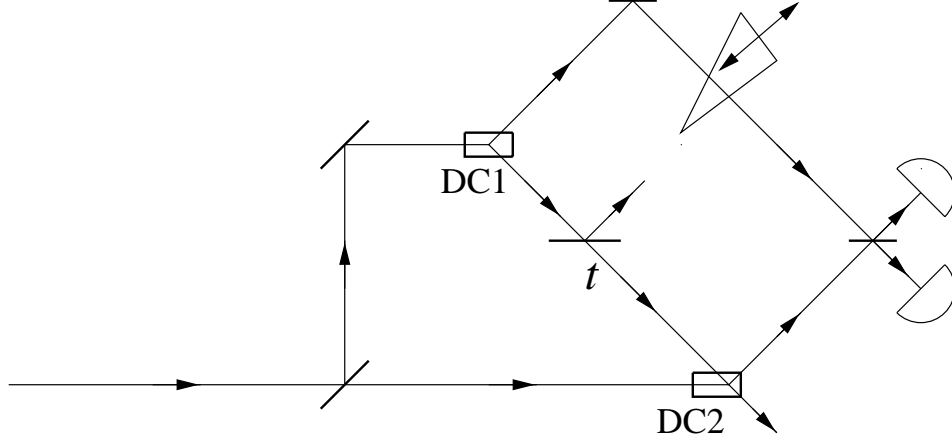


FIG. 1. Experimental setup applied and analyzed by WZM. Two downconverters, DC1 and DC2, pumped by a light from a common source are aligned so that the idler photon from DC1 is injected into DC2 after transmission through a beamsplitter with transmission amplitude  $t$ . The signal photons from the downconverters are combined by another beamsplitter, and an interference signal is recorded as function of the variation in path length of one of the signal fields.

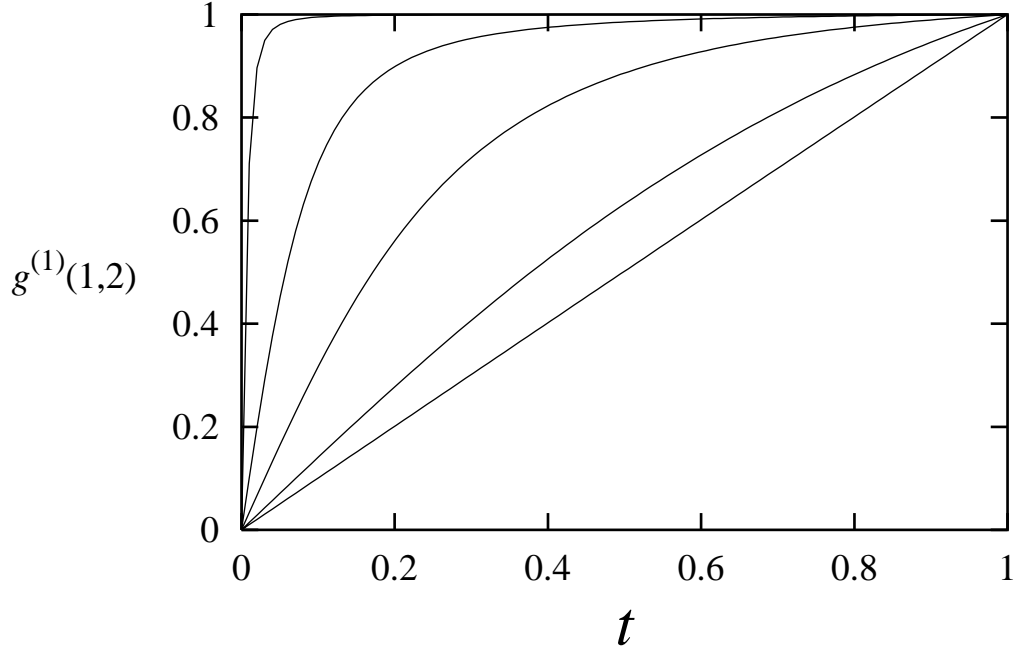


FIG. 2. First order mutual coherence function  $g^{(1)}(1,2)$  of the two signal fields, observable as the maximum fringe visibility in the interference signal recorded in the set-up of Fig.1. The theoretical expression (11) is shown as function of the transmission amplitude  $t$  for different values of  $\bar{n}_1$ , the mean photon number entering the beam splitter in Fig. 1. Specifically, from the lowest to the highest curve,  $\bar{n}_1 = 10^{-2}, 1, 10, 100, 10^4$ .